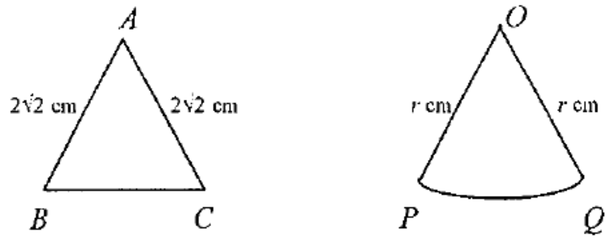
CORE MATHEMATICS (C) UNIT 2 TEST PAPER 7

1. Find the remainder when $(x^4 - 3x^3 + 2x - 1)$ is divided by $(x + 1)$. [3]

2. The diagram shows an isosceles triangle ABC whose equal sides are $2\sqrt{2}$ cm long and a sector OPQ of a circle of radius r cm. Angle BAC = angle $POQ = \dfrac{\pi}{4}$ radians.



Given that the triangle and the sector have equal areas, find r correct to 2 decimal places. [4]

3. (i) Find $\int (3x - 1)^2 \, dx$. [4]

   (ii) Hence write down *two* possible functions $f(x)$ such that $f'(x) = (3x - 1)^2$. [2]

4. In the binomial expansion of $\left(2x + \dfrac{1}{2x}\right)^6$ in descending powers of x, find in their simplest form

   (i) the first two terms, [4]

   (ii) the term which is independent of x. [3]

5. In the triangle XYZ, $XY = (x + 1)$ cm, $YZ = x$ cm and $XZ = y$ cm. Angle $XYZ = 120°$.

   (i) Show that $y^2 = 3x^2 + 3x + 1$. [3]

   (ii) When $x = 7$, find the size of angle XZY, in degrees to 1 decimal place. [4]

6. (i) A car is bought for £12,000. Each year, its value decreases by 15%.
   Show that the values of the car in successive years form a geometric sequence, and find the value of the car after 12 years, to the nearest pound. [4]

   (ii) Find the exact value of $\sum_{r=1}^{15} (7 - 3r)$. [3]

7. (i) Solve for x the equation $2 \log_{10} 5 + \log_{10}(10 - x) = 3 + \log_{10} x$, giving your answer as an exact fraction. [5]

   (ii) By substituting $y = 2^x$, or otherwise, find to 1 decimal place the value of x for which
   $$2^{2x+1} - 2^{x+2} = 6.$$ [5]

8. (i) Find, to 1 decimal place, the values of x between -180 and 180 for which
   $$\cos 2x^\circ = 0.35.$$ [3]

   (ii) Given that $\sin\theta = \dfrac{15}{17}$ and $-180^\circ < \theta < 180^\circ$, find the possible values of
   (a) $\cos\theta$, (b) $\tan\theta$. [4]

   (iii) Show that the equation $3\cos^2 x = 2(\sin x + 1)$ can be written as a quadratic equation in $\sin x$. Hence solve this equation for $0 \le x \le 2\pi$, giving your answers in radians to two decimal places. [5]

9. The equation of a curve C is $y = x^3 - 4x^2 + 5x - 2$.

   (i) Show that $y = 0$ when $x = 1$. Hence express y as the product of three linear factors. [4]

   (ii) Calculate the coordinates of the turning points of C, identifying each as a maximum or a minimum. [5]

   (iii) Sketch the curve C, clearly showing the turning points and any points of intersection with the axes. [3]

   (iv) Find the area of the finite region contained between C and the x-axis. [4]

CORE MATHS 2 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1.	Remainder = $f(-1) = 1 + 3 - 2 - 1 = 1$	M1 A1 A1	3
2.	$4 \sin \pi/4 = \frac{1}{2} r^2 (\pi/4)$ $\quad r^2 = 16\sqrt{2}/\pi$ $\quad r = 2.68$	M1 A1 M1 A1	4
3.	(i) $\int (9x^2 - 6x + 1)\,dx = 3x^3 - 3x^2 + x + c$	M1 A1 A1 A1	
	(ii) e.g. $3x^3 - 3x^2 + x$, $3x^3 - 3x^2 + x + 1$	B1 B1	6
4.	(i) $\left(2x + \frac{1}{2x}\right)^6 = (2x)^6 + 6(2x)^5\left(\frac{1}{2x}\right) + \ldots = 64x^6 + 96x^4 + \ldots$	M1 A1 M1 A1	
	(ii) Term is $\binom{6}{3}(2x)^3\left(\frac{1}{2x}\right)^3 = 20$	M1 A1 A1	7
5.	(i) By cosine rule, $y^2 = x^2 + (x+1)^2 - 2x(x+1)(-1/2) = 3x^2 + 3x + 1$	M1 M1 A1	
	(ii) When $x = 7$, $y^2 = 169$ so $y = 13$ $\quad \sin Z / 8 = \sin 120° / 13$	B1 M1	
	$\sin Z = 0.533$ $\quad$ Angle $XZY = 32.2°$	M1 A1	7
6.	(i) Each year's value is 0.85 times the previous years, hence geometric	B2	
	£12,000 x 0.85^{12} = £1707	M1 A1	
	(ii) $4 + 1 + \ldots + (-38) = 15(4 - 38)/2 = 15(-17) = -255$	M1 A1 A1	7
7.	(i) $\log_{10} 25(10 - x) = \log_{10} 1000 + \log_{10} x$ $\quad 250 - 25x = 1000x$	M1 A1 A1	
	$1025x = 250$ $\quad x = 10/41$	M1 A1	
	(ii) $2y^2 - 4y - 6 = 0$ $\quad 2(y+1)(y-3) = 0$	M1 A1 A1	
	$2^x = -1$ (impossible) or $2^x = 3$ $\quad x = \log_2 3 = 1.6$	M1 A1	10
8.	(i) $2x = \pm 69.51, \pm 290.49$ $\quad x = \pm 34.8, \pm 145.2$	M1 A1 A1	
	(ii) Using 8, 15, 17 triangle, (i) $\cos\theta = \pm 8/17$, (ii) $\tan\theta = \pm 15/8$	M1 A1 M1 A1	
	(iii) $3(1 - \sin^2 x) = 2(\sin x + 1)$ $\quad 3\sin^2 x + 2\sin x - 1 = 0$	M1 A1	
	$(3\sin x - 1)(\sin x + 1) = 0$ $\quad \sin x = 1/3$ or -1 $\quad x = 0.34, 2.80, 4.71$	M1 A1 A1	12
9.	(i) When $x = 1$, $y = 1 - 4 + 5 - 2 = 0$	B1	
	$y = (x-1)(x^2 - 3x + 2) = (x-1)(x-1)(x-2)$	M1 A1 A1	
	(ii) $dy/dx = 3x^2 - 8x + 5 = (3x - 5)(x - 1) = 0$ when $x = 1$, $x = 5/3$	M1 M1 A1	
	T.P.s are (1, 0) maximum, (5/3, –4/27) minimum	A1 A1	
	(iii) Curve sketched, cutting y-axis at (0, –2) and *touching* x-axis at (1, 0)	B1 B1 B1	
	(iv) $\int_1^2 y\,dx = \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{5x^2}{2} - 2x\right]_1^2 = \frac{15}{4} - \frac{28}{3} + \frac{15}{2} - 2 = -\frac{1}{12}$ Area $= \frac{1}{12}$	M1 A1 M1 A1	16

